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Design Sensitivity Analysis of an Angular Rate Sensor

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ABSTRACT: The paper is concerned with sensitivity analysis of a rotational motion sensor that uses Coriolis effect and a vibrating quartz tuning fork to sense angular rate. The finite element model is employed to both studying the system's measurement performances and appreciation of the sensitivity of these to the various influencing effects. The performance of computations has been enhanced by means of a dynamically reduced model based on truncation of dynamic contributions of higher modes of a non-rotating structure. The indeterminacy of the measurement results due to the change in dynamic properties resulting from the deviations of resonance frequencies caused by geometrical errors has been studied. The sensitivity functions have been derived by using gradient technique basing upon the finite element model.

Keywords: Sensors, sensitivity analysis, finite element modelling, dynamic reduction .

1 Introduction

In the analysis of measurement systems we are concerned with many aspects of their behaviour: response to input signals, response to disturbances, and the sensitivity of these to parameter changes. One of the foremost problems in measurement systems analysis and synthesis is that of the reduction of sensitivity of the systems response to the parameter variations attributable to the tolerances allowed to the device. Sensitivity analysis should be undertaken by the design-oriented methods. This implies the use of higher level of abstraction models that deal not only with fundamental principles underlying measurement systems but also with principles of treatment of deviations and uncertainties based on the analysis of instrument structure and the effects of sensitivity to parameters variations and external influences. In many cases the models that are formed on basic physical theory and phenomena of the relevant systems should be completed by more comprehensive consideration.

2 Angular Rate Sensor and its Model

We consider a computational model of the GyroChip family sensor that uses a micromachined quartz element - a vibrating quartz tuning fork - to measure angular rotational velocity [1, 2].

Using the Coriolis effect, the rotational motion about the sensor's longitudinal axis produces a DC voltage proportional to the rate of rotation. The description and performance specifications of the sensor are available [1, 2]. The sensor representation given [1, 2] allows definition of organic components of a sensor and describes their qualities and interactive behaviour.

Having determined a physical effect and the possibility of a sensing technique, adequately formulated models enable existing systems to be studied in modes of operation in which they may be called to provide and allow the modelling process to better simulate the system by providing numerical understanding.

The dynamic equation of the finite element of the tuning fork is obtained as [3, 4]

$$[M]\{\ddot{U}\} + 2\omega[C_1]\{\dot{U}\} + \left([K] - \omega^2[K_1] + \varepsilon[K_2]\right)\{U\} = \\ = \{R\} + \{F\} + \omega^2[K_1]\{X\} - \varepsilon[K_2]\{X\}; \qquad (1)$$

where [M], [C], [K], $[K_1]$, $[K_2]$, matrices of the element, {U}- nodal displacement vector, {F} - nodal excitation forces caused by the piezoelectric effect, {X} – vector of nodal coordinates of the finite element.

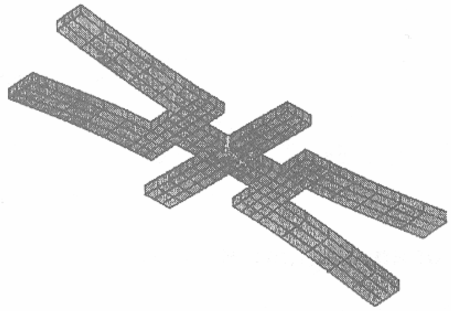
The quartz tuning fork has a number of modes of vibration from which actually only four essentially different shapes (displayed on Fig. 1) are of interest for the angular velocity meter application.

3 Design Sensitivity Analysis

As the tuning fork of the angular velocity meter is a mechanical vibrating system with high value of the mechanical Q-factor, its dynamic features and performance depends significantly upon natural frequencies and shapes of vibration. The influence of the design parameters can be most effectively carried out by employing gradient techniques.

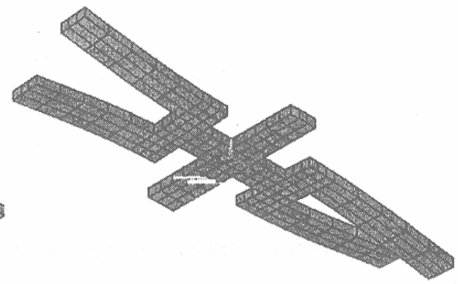
The finite element matrices of the fork can be presented as functions $[K(b_i)],[M(b_i)]$ of the geometric parameters b_i of the structure. The relations between small variations of design parameters and corresponding variations of natural frequencies of vibration are obtained by using the free vibration equation

$$\left([K] - \omega^2[M]\right)\{U\} = 0 \ . \qquad (2)$$



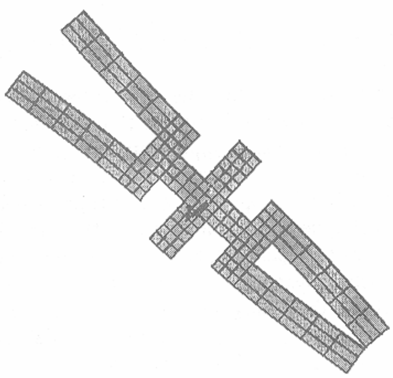
Mode 3, out-of-plane,
Natural frequency 8110Hz

(a)



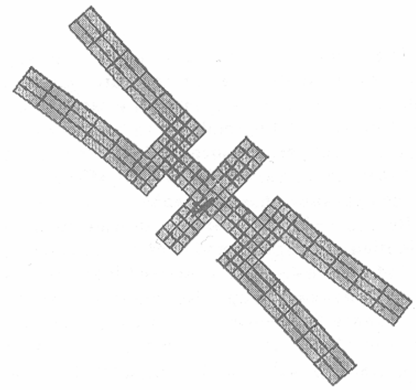
Mode 4, out-of-plane,
Natural frequency 8113Hz

(b)



Mode 8, in-plane
Natural frequency 25398Hz

(c)



Mode 9 , in-plane,
Naural frequency 25400Hz

(d)

Fig. 1

Sensitivity is defined in terms of a sensitivity function, which denotes the sensitivity of the system design decisions to variations in the system parameters. The first variation of (2) gives the following relations:

$$\partial\{\zeta_i\} = [C_i]\partial\{b\} \quad , \qquad (3)$$

where $[C_i]=\{y_i\}^T\left(\frac{\partial[K]}{\partial\{b\}}-\zeta_i\frac{\partial[M]}{\partial\{b\}}\right)\{y_i\}$ is the matrix of sensitivity coefficients, $\zeta_i=\omega_i^2$ - square of the i-th angular natural frequency, $\{y_i\}$ - the vector describing the i-th shape of vibration.

The sensitivity functions obtained can be further used to bring about modifications needed in the structure's dynamic properties and determine which modifications would be the most effective for the desired change.

The analysis of obtained sensitivity coefficients indicates that the stiffness of the supporting bar described by its length and width has the main influence upon the difference of the 3[rd] and 4[th] natural frequencies. The plot of the values of the two frequencies against the value of parameter l (half length of the supporting bar) is presented in Fig.2. At parameter value $l \approx 1.54mm$ natural frequencies of 3[rd] and 4[th] modes are equal, and this requires a special attention in dynamic analysis as here the magnitudes of the two natural frequencies may counterchange as a result of small variation of the

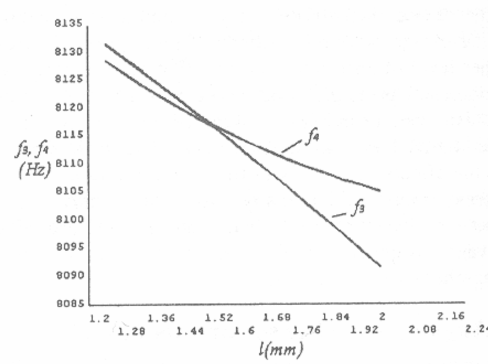


Fig.2

geometric parameter l.

4 Reduction of Dynamic Equations

In case of moderate velocities of rotation of the frame the modal coupling and resonance phenomena of the fork in the rotating frame can be much better understood by expressing the equations in modal coordinates of the non-damped and non-rotating structure, the vibration modes of which have a clear interpretation.

Moreover, the aim of having the modal properties of a non-rotating structure as target functions in geometrical design of the fork is seen as more natural and convenient.

We denote by $\omega_1, \omega_2, \ldots \omega_n$ the natural frequencies of the fork and by [Y] the matrix containing in its columns the natural forms of vibration of a non rotating fork. By neglecting the effects caused by angular acceleration and centripetal forces, the steady vibration of the fork in the non-rotating frame is governed by the equation in terms of modal displacements $\{z\}$ as

$$\{\ddot{z}\}+\left[diag(\mu)\right]\{\dot{z}\}+\left[diag(\omega^2)\right]\{z\} = -2\Omega\left[Y\right]^T\left[G\right]\left[Y\right]\{z\}+\left[Y\right]^T\{F\}, \qquad (4)$$

where $\mu_1, \mu_2, \ldots \mu_n$ the modal damping coefficients obtained as $\mu_i = \alpha + \beta\omega_i^2$ and the substitution $\{U\} = [Y]\{z\}$ and relations $[Y]^T[M][Y] = [I]$, $[Y]^T[K][Y] = [diag(\omega^2)]$ have been employed. Though in modal coordinates, the equations are still coupled as the matrix $[Y]^T[G][Y]$ is non-diagonal.

Further simplification of the equations can be carried out by neglecting the dynamic contributions of higher modes of the fork. We partition the modes and modal displacements into two sets so that the displacement vector can be presented as $\{U\} = [Y_1]\{z_1\} + [Y_2]\{z_2\}$, and truncate the terms corresponding to inertial and damping forces of the second modal set. Finally the following equation in terms of modal displacements of only first set is obtained as

$$\{\ddot{z}_1\}+\left(\left[diag(\mu_1)\right]+2\Omega\left[Y_1\right]^T\left[G\right]\left[Y_1\right]\right)\{\dot{z}_1\}+ +\left[diag(\omega_1^2)\right]\{z_1\} = -2\Omega\left[Y_1\right]^T\left[G\right]\left[S_k\right]\{\dot{F}\}+\left[Y_1\right]^T\{F\}; \qquad (5)$$

where

$[S_k] = [K]^{-1} - [Y_1]\left[diag(1/\omega_1^2)\right][Y_1]^T$ is the higher modes compliance matrix.

The model consisting of piezoelectric shell elements has been programmed in ANSYS and FOTRAN. The reason of application several different programming environments was that the ANSYS program does not allow obtaining the harmonic vibration response of the rotating structure. Therefore the structural stiffness and mass matrices have been printed to files and taken into a FORTRAN program that was written to perform

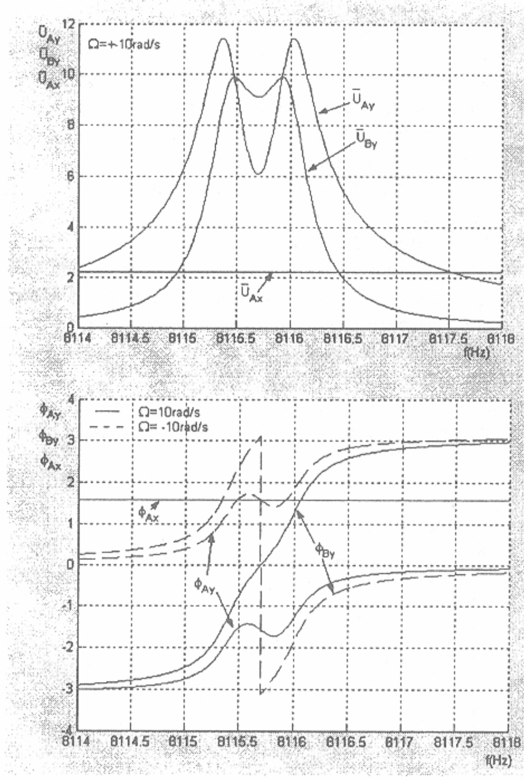


Fig.3

calculations by using the above-presented dynamically reduced model.

The accuracy of the solution depends upon the number of modes taken into consideration. We always can obtain the exact solution when solving equations with all modes participating with their dynamic contributions. If the solution is close to the exact one with taking into account the dynamic contributions of only few modes, the participating modes are decisive for the operation law of the angular frequency meter.

The detailed analysis of the influence of modes by adding them one by one to the dynamic model leads to the conclusion that the 9x9 reduced dynamic model obtained by taking into account the dynamic contributions of only first 9 modes is accurate enough and can be used instead of the full 1326x1326 original model. It can be noticed that 8^{th} and 9^{th} dynamic modal contributions though excited far below the resonance are important for proper representation of the dynamic features of the system. The time savings during calculations of amplitude-frequency and phase-frequency curves of the model are really impressive.

5 Analysis of the Dynamic Behaviour

In order to perform its function as the sensitive element of the angular frequency meter, the fork is excited by means of the applied electric voltage over one half of the fork (input tines). The frequency of excitation of the fork is close to the natural frequencies of resonant out-of-plane modes 3 and 4 (Fig.1), though the modes are not excited because of the in-plane action of electromechanical excitation forces. For in plane vibration, the excitation frequency is far below the resonance, so they may be regarded as non-resonant.

In rotating frame, in-plane vibration of input tines excites both neighbouring modes 3 and 4. The vibration law of output tines depends upon the mutual position of values of natural frequencies f_3 and f_4 on the frequency axis.

If $f_3 = f_4$, the out-of-plane vibration of output tines will not be excited because of eliminating each other contributions of modes 3 and 4. However, the resonant out-of-plane vibration will be excited in input tines. This mode of operation is based on a very narrow allowable frequency range in order to keep the output vibrations essentially on

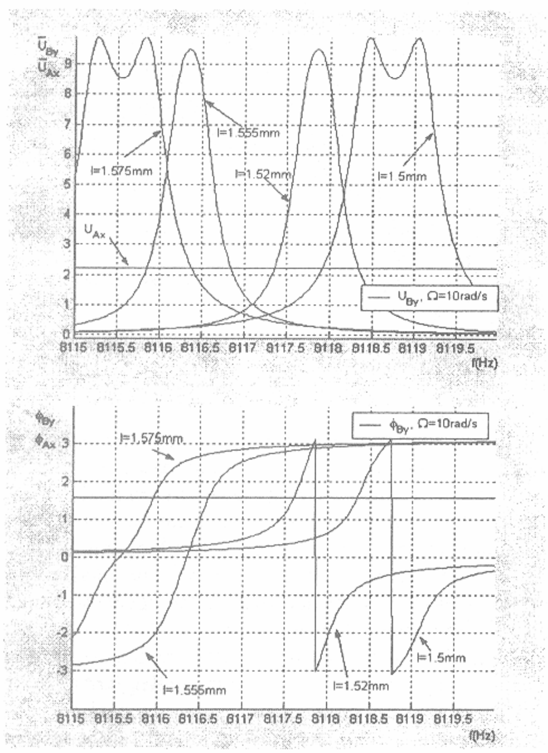


Fig.4

the peak of AFCH curve. Similar dynamic properties can be obtained in case when the fork is excited over all the surface of the fork. In a rotating frame, the in-plane vibration would excite only mode 3 of out-of-plane vibration;

The optimum separation of natural frequencies f_3 and f_4 by selecting proper geometrical parameters allows obtaining out-of-plane vibration of output tines, the AFCH of which has a plateau or a local minimum on its top. The tolerance of the excitation frequency is allowable in wider range, Fig.3, where $\bar{U}_A, \bar{U}_B$ denote the amplitudes of vibration of input and output tines.

If f_3 is very close, but not equal to f_4, the out-of-plane vibration of output tines will be excited by rotation of the frame. The phase of vibration of output tines will depend upon the mutual positions of natural frequencies of symmetrical (f_3) and anti-symmetrical (f_4) out-of-plane modes. As the frequency values of them interchange (i.e., f_3 becomes greater as f_4), the phase of vibration of output tines changes through value π, Fig.4.

It means, the effect is the same as the change of the sign of angular frequency of rotation of the frame and can lead to misinterpretation of the direction of the rotation velocity.

4 References

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